Title: How High Will Your Ball Bounce?

Brief Overview:

The students will investigate, create and analyze data using the TI-83 plus calculators. Students will enter real-world data into lists, then graph and analyze the data. Students will also do an experiment that will produce approximately linear data. Students will do a linear regression and make predictions using their regression equations. If time permits, they will test out their predictions.

NCTM Content Standard/National Science Education Standard:

Data Analysis and Probability

Students will formulate questions and collect, organize, and display relevant data to answer them. They will determine regression equations that model the data using appropriate technology. The students will also make inferences and predictions that are based on the data. They will determine regression equations and discuss the appropriateness of these equations as they relate to their data, The students will use bivariate measurement data, be able to display a scatter plot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools.

Algebra

Students will be able to use mathematical models to represent a physical situation and analyze changes in the situation. Students will represent data using tables and graphs. They will develop and evaluate inferences and predictions that are based on data. They will interpret representations of functions of two variables. The students will judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

Science as Inquiry

Students will participate in scientific investigations and formulate scientific explanations. They will reflect on the concepts that guide the inquiry. Students also will analyze evidence and data and communicate their explanations.

Links to Maryland High School Mathematics Core Learning Goals:

- **1.1.1**
  The student will recognize, describe, and/or extend patterns and functional relationships that are expressed numerically, algebraically, and/or geometrically.

- **1.1.2**
  The student will represent patterns and/or functional relationships in a table, as a graph, and/or by mathematical expression.

- **3.1.1**
  The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

- **3.2.2**
  The student will interpret data and/or make predictions by finding and using a line of best fit
  .

Grade/Level:

Grades 9-12

Duration/Length:

Two 45-minute class periods for lessons 1 and 2. Lesson 3 will require at least parts of two class periods.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Identify dependent and independent variables for a data set.
- Construct a scatter plot on graph paper.
- Understand functions written in slope-intercept form.
- Recognize patterns in plotted data.

Student Outcomes:

Students will:

- enter data into lists using the TI-83 plus graphing calculator.
- create a scatter plot using the TI-83 plus graphing calculator.
- create a linear regression model using the TI-83 plus graphing calculator.
- make predictions about data using the TI-83 plus graphing calculator.
- perform an experiment to collect data.
- understand the use of correlation coefficients.
- identify outliers for a data set.

Materials and Resources:

- TI-83 plus graphing calculator
- Small rubber balls
- Tape measures, preferably cloth
- Graph paper
- Masking tape

Development/Procedures:

Lesson 1

Preassessment – The teacher will begin the lesson by distributing an opening activity worksheet (Opening Activity Day 1 Worksheet). The purpose is to determine if students can graph data and recognize a linear pattern.

Launch – The teacher will show students how to enter data and calculate a linear regression model by entering Opening Activity Day 1 information into lists on the TI-83 plus calculator. The teacher should have students compare their calculator's graph with the graph they did on the worksheet.

Teacher Facilitation – The teacher should distribute Olympic Data to students. Data are in inches and contain the winning distance for High jump, Discus, and Long jump for the years 1900 through 1984. The teacher should have students use **Year** and **Long jump** as their independent and dependent variables. The independent variable, year data, should be entered into $\mathbf{L_1}$ using 1900 as the base year, i.e., $\mathbf{1900 \Rightarrow 0}$. Students should enter data into their lists and create a scatter plot. They should decide if their plot has a linear pattern, and if so use the calculator to determine the linear regression model. Students should use their model to predict a winning distance for 1944 and 2040. Using the model, students should decide the best estimate for the Olympic year during which the winning long jump distance would have been approximately 308 inches. (This can also be done for 293 inches.)

The teacher should discuss the meaning of correlation coefficient (r value). From the correlation coefficient you can determine the direction of the association and its strength. The teacher may wish to use descriptors such as Very Good, Good or Moderate to describe the strength of the association. For this activity we suggest assigning numerical values of:

Very Good	$\mathbf{\lvert r \rvert \geq .95}$
Good	$\mathbf{.90 \leq \lvert r \rvert < .95}$
Moderate	$\mathbf{.75 \leq \lvert r \rvert < .90}$

The correlation coefficient also tells the direction of the association. If "r" is positive the data has a positive association. If "r" is negative the association is negative. The teacher should also mention the term outlier. For this activity we suggest using a general definition of a value that appears to be too large or too small with respect to the rest of the data. For this data set the winning distance of 350.5 inches in 1968 is an outlier. The teacher may wish to have students investigate the results from that year.

Student Application – The teacher should distribute the data for the Boston Marathon Women's Winning Times From 1972 – 1984. (We suggest using an overhead transparency.) Using 1970 as the base year, have students enter the data and create a scatter plot. If the data appears to be linear, they should calculate the linear regression model. They should use their model to predict the winning time in 1990. After completing this assignment the teacher should have the students add the winning times

from 1985 – 1997 to their list. The students should create a new scatter plot and determine if it still appears to be linear. They should also compare their predicted 1990 time with the actual winning time. The teacher should use this to emphasize that predictions should be made for missing data within the given data range. Students should be directed to return to their Olympic long jump predictions for 1944 and 2040 and asked which prediction would be more reliable.

Embedded Assessment – The teacher should check each student's progress in completing the activities.

Reteaching/Extension – For a home assignment the students should be directed to enter the Olympic high jump data into their calculator.
They should:

- create a scatter plot, and identify any outliers.
- calculate the linear regression model.
- use the model to predict the winning height for an **appropriate** year.
- use the model to estimate the Olympic year during which the winning throw would have been approximately 2,110 inches.
- determine the correlation coefficient and strength of the association.

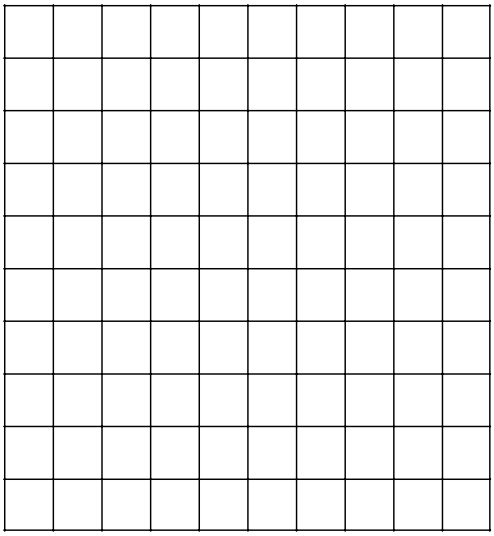
Name: _____

Opening Activity Day 1

Create a Scatter Plot for the following data.

1.

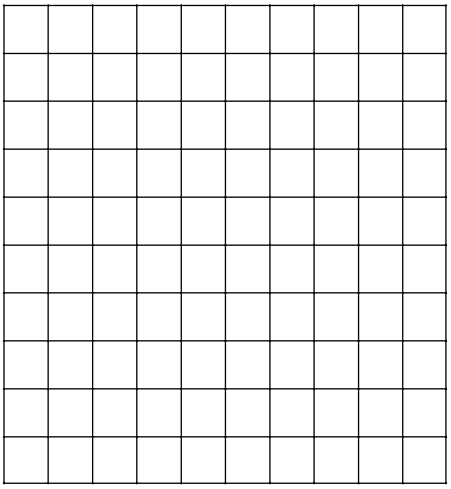
X	Y
2	5
3	7
4	8.5
6	12
7	16
7.5	17.5
8	18



Does the Scatter Plot appear to be approximately linear? _____

2.

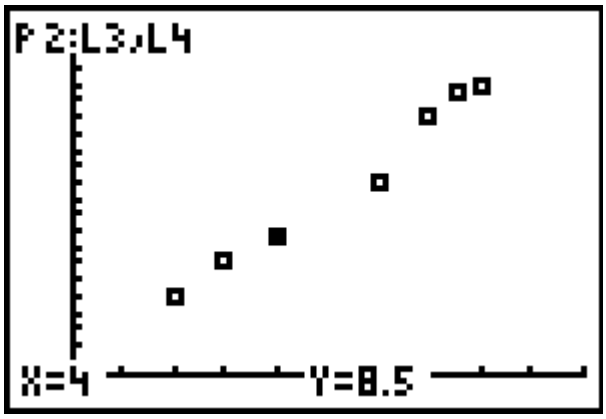
X	Y
1	6
2	1
3	4.5
3	7
4.5	4.5
5	9
6.5	1.5



Does the Scatter Plot appear to be approximately linear? _____

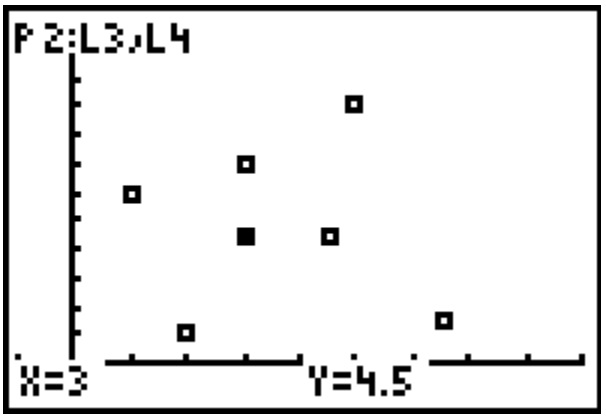
ANSWERS

1.



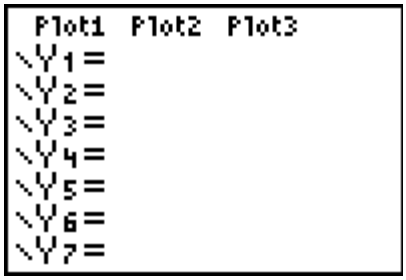
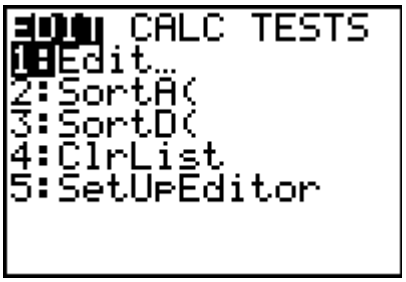
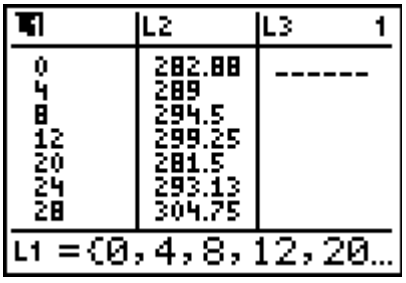
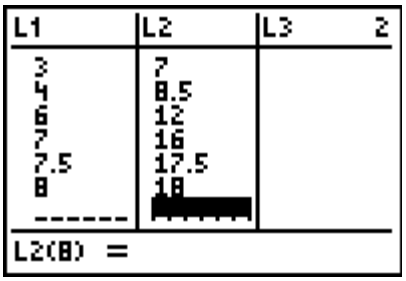
Linear? YES

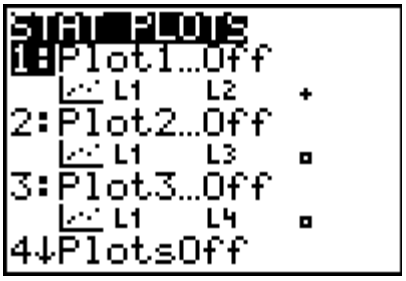
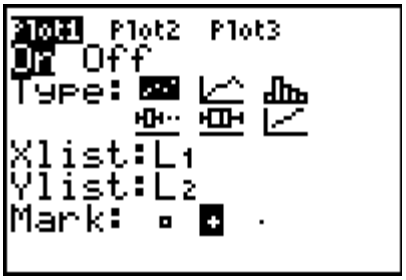
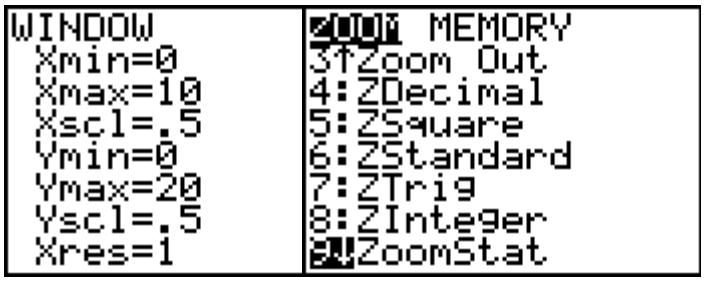
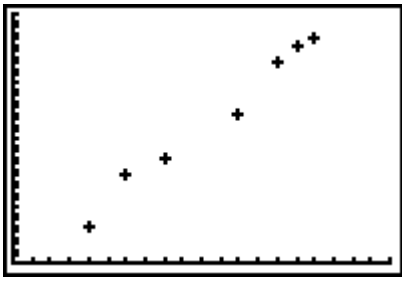
2.



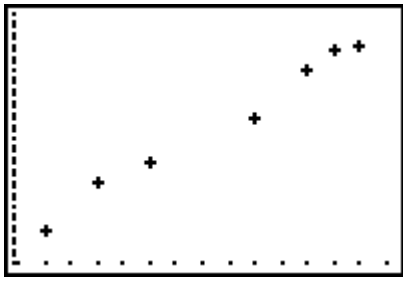
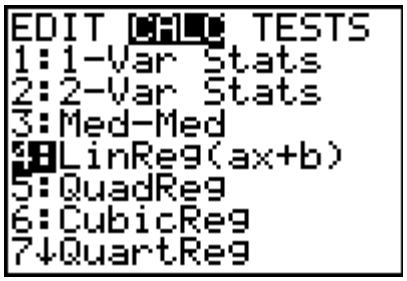
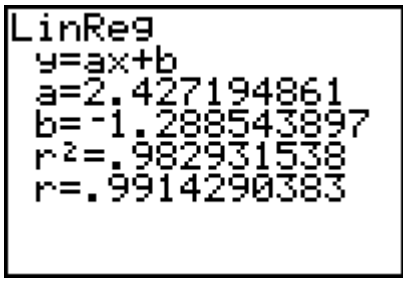
Linear? NO

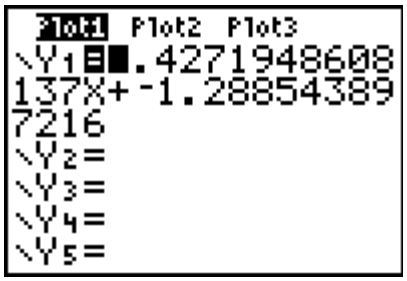
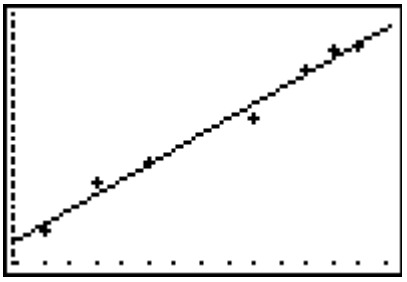
Instructions for Creating a Scatter Plot
On the TI-83 Plus Graphing Calculator

Procedure	TI-83 + Screen
Open the **Y =** screen, and check that no plots are entered. If the first row (Plot1, Plot2, or Plot3) is highlighted, use the up arrow to move your cursor over the highlighted plot and click **ENTER**. If there is an equation next to "\Y1=", move your cursor to the right of the "=" and press **CLEAR**.	
Enter **STAT**. Highlight 1:Edit. Hit **ENTER**.	
If there is data in your lists, press the up arrow until L1 or L2 is highlighted. Then hit **CLEAR** and **ENTER**. If you are missing a list, ask your teacher to fix it with SetUpEditor.	
Enter the independent data (X) into L1. Enter the dependent data (Y) into L2.	

Procedure	TI-83 + Screen
Press **2nd** **Y =** to go to STAT PLOT.	
Hit **ENTER** . Choose Plot1 On, Type: Scatter plot, Xlist: L_1, Ylist: L_2, and either Mark.	
Now press **WINDOW** . By examining your data, choose appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl.. Alternatively, you can choose **ZOOM** 9.	
Now you can **GRAPH** your scatter plot.	

Instructions for Creating a Linear Regression on the TI-83 Plus Graphing Calculator

Procedure	TI-83 + Screen
Turn diagnostics on by clicking **2nd** **0** to view the CATALOG. Then scroll down with the down arrow until **Diagnostics On** . Press **ENTER** to exit catalog and **ENTER** to turn on diagnostics.	
Clear your equations, enter the data in L_1 and L_2, and scatter plot your points, as described in **Instructions for Creating a Scatter Plot On the TI-83 Plus Graphing Calculator.**	
Press **STAT** , press the right arrow ▷ to **CALC**, and choose **4** 4:LinReg(ax+b). (Option 8 also produces a linear regression model, but a and b values are interchanged.)	
"LinReg(ax+b)" is displayed on your screen. Click **2nd** , **1** (to enter L1), **,** , **2nd** , **2** (to enter L2), **,** , **VARS** , ▷ , **1** , **1** (to enter Y_1). This specifies that the independent variable is stored in L1, the dependent variable is stored in L2, and the linear regression model will be stored in Y1.	
Press **ENTER** to calculate the equation of the linear regression model. The correlation coefficient **r** tells you the direction and strength of the association.	

Procedure	TI-83 + Screen
Click **Y =** . The linear regression equation should appear as Y_1.	
Press **GRAPH** , and you will see the scatter plot along with the regression line.	

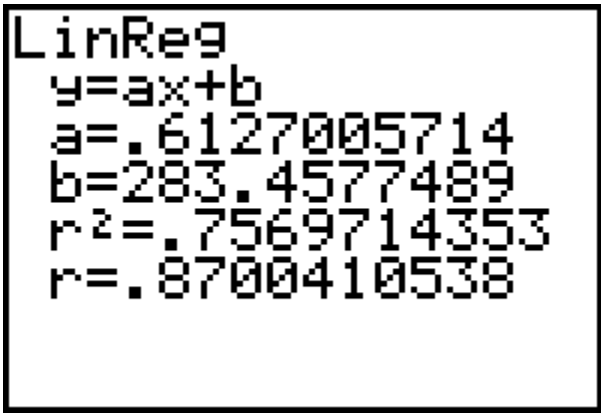
OLYMPIC DATA

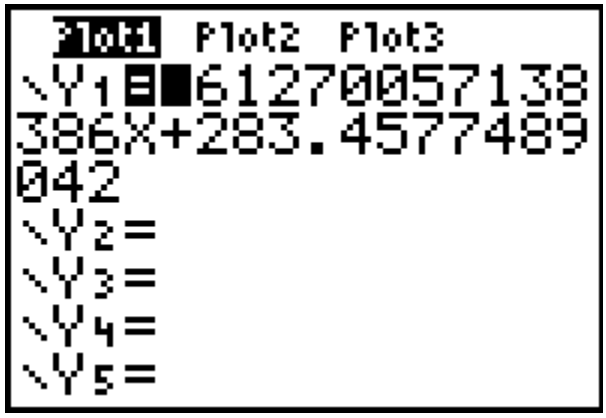
This information was obtained from: www.statsci.org/data/general/olympic.html.

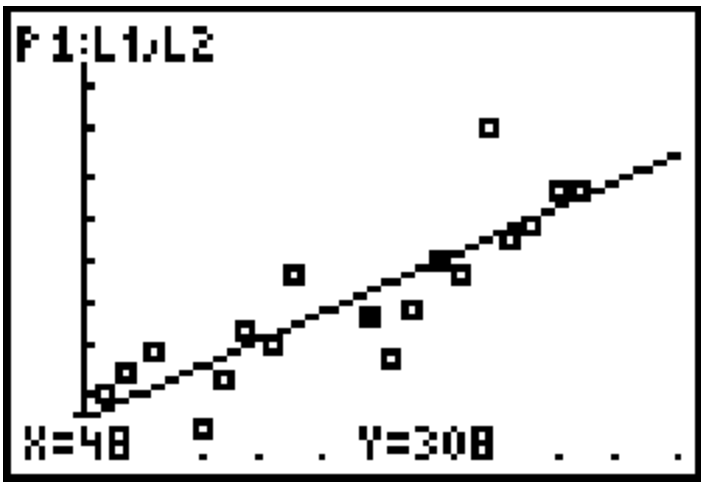
High jump	Discus throw	Long jump	Year
74.8	1418.9	282.875	1900
71	1546.5	289	1904
75	1610	294.5	1908
76	1780	299.25	1912
76.25	1759.25	281.5	1920
78	1817.125	293.125	1924
76.375	1863	304.75	1928
77.625	1948.875	300.75	1932
79.9375	1987.375	317.3125	1936
78	2078	308	1948
80.32	2166.85	298	1952
83.25	2218.5	308.25	1956
85	2330	319.75	1960
85.75	2401.5	317.75	1964
88.25	2550.5	350.5	1968
87.75	2535	324.5	1972
88.5	2657.4	328.5	1976
92.75	2624	336.25	1980
92.5	2622	336.25	1984

All data is in inches.

Long Jump Linear Regression Model







Predicted distance for 1944 is 310.417 inches (rounded to nearest thousandth).

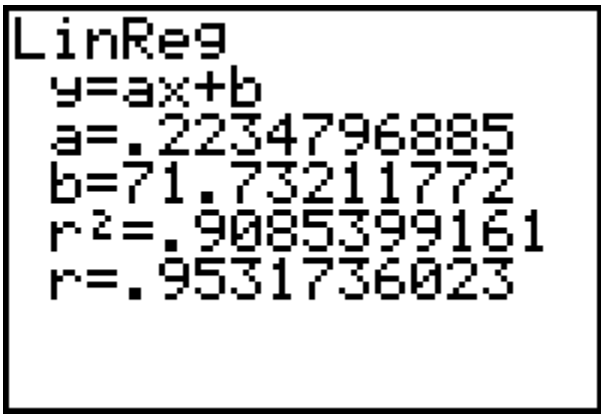
Predicted distance for 2040 is 369.236 inches (rounded to nearest thousandth).

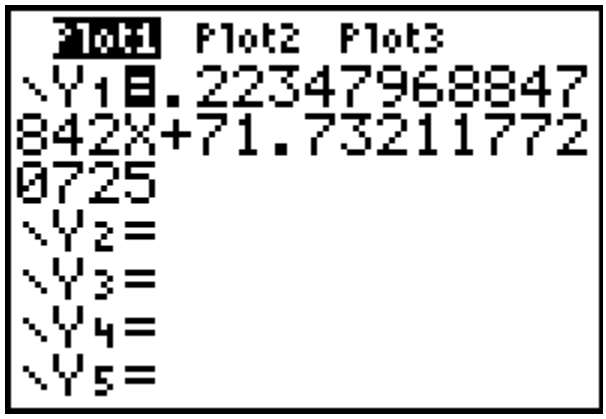
The strength of the association is moderate.

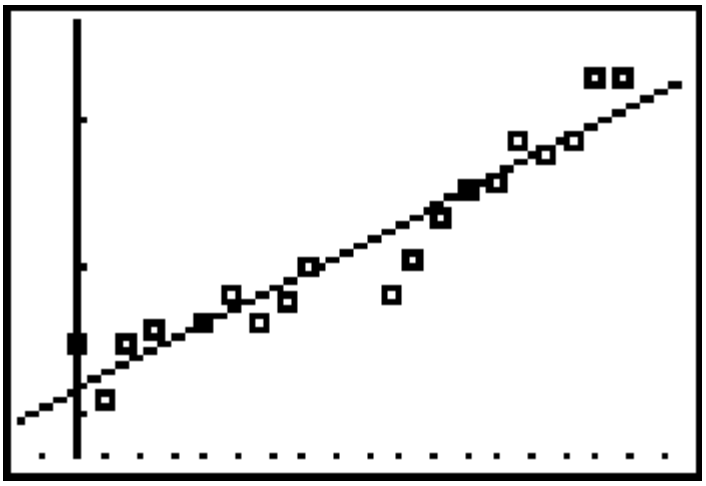
Distance of 308 inches is for the Olympic year 1940.

Distance of 293 inches is for 1916.

High Jump Linear Regression Model







NO Obvious Outliers

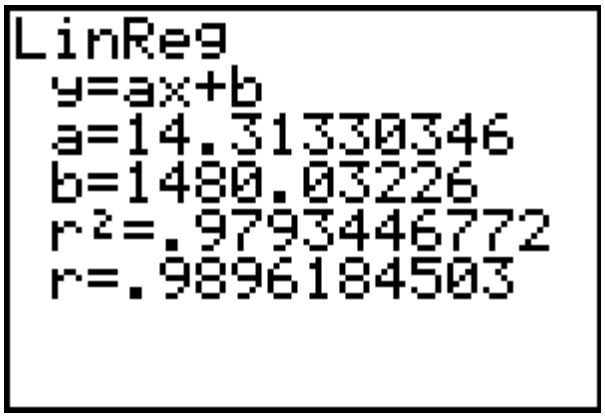
Predicted height in 1916 is 75.308 inches.

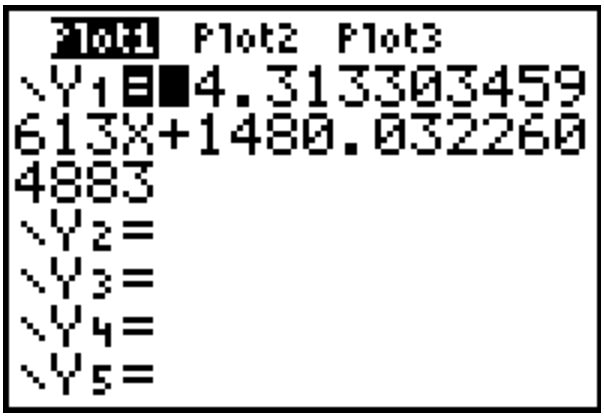
Predicted height in 1940 is 80.671 inches.

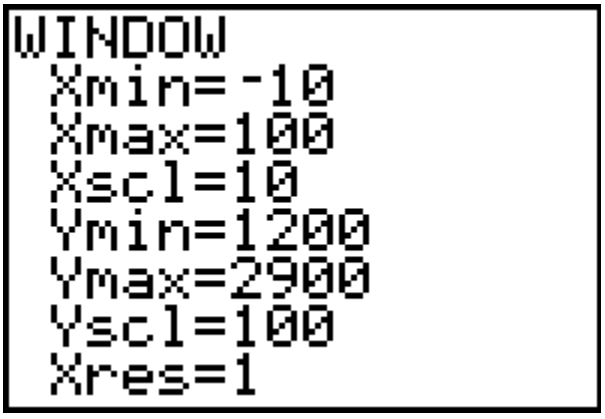
Predicted height in 1944 is 81.565 inches.

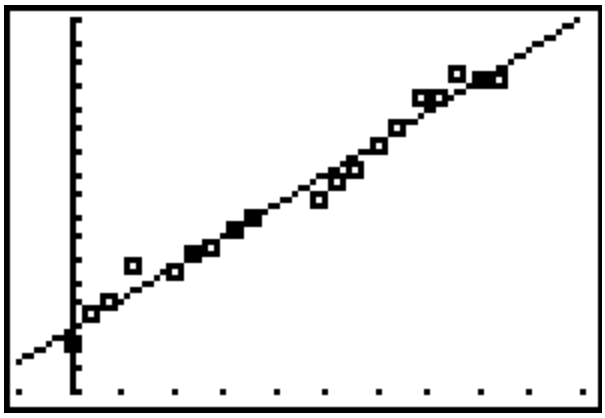
The strength of the association is very good.

Discus Linear Regression Model









NO Obvious Outliers

Predicted distance for 1916 is 1709.045 inches.

To get the prediction, I substituted 16 into my regression model for "x", then determined the "y" value.

To use the calculator to get this answer, the student could go to 2nd CALC, value then enter 16. You could also go to 2nd TBLSET, TBLStart = 16, then 2nd TABLE.

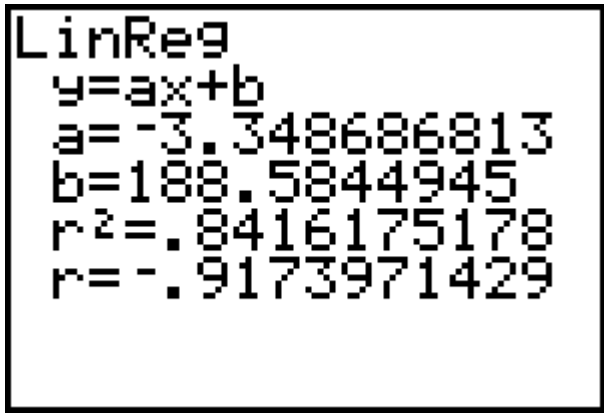
Distance of 2,110 inches in 1940.

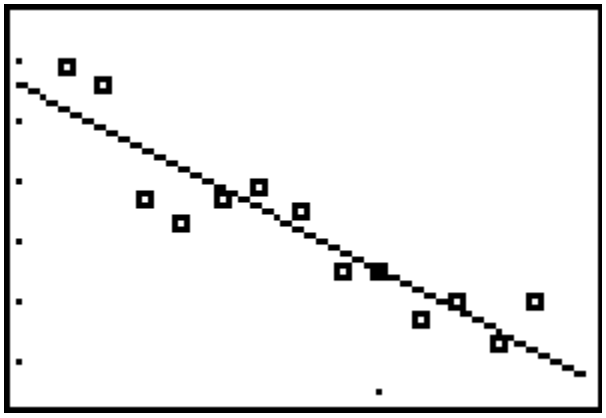
The strength of the correlation is very good.

Boston Marathon Women's Winning Times From 1972 – 1984
(time in minutes)

Year	Winning Time
1972	188.967
1973	185.983
1974	167.183
1975	162.400
1976	167.167
1977	168.550
1978	164.867
1979	155.250
1980	154.467
1981	146.767
1982	149.550
1983	142.717
1984	149.467

Linear Regression Model 1972 - 1984:

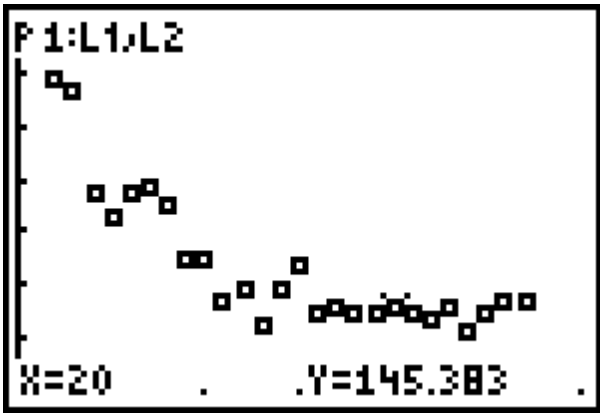




Predicted Winning Time for 1990 is 121.61 minutes.

Boston Marathon Women's Winning Times From 1985 – 1997

Year	Winning Time
1985	154.100
1986	144.917
1987	145.350
1988	144.500
1989	144.550
1990	145.383
1991	144.300
1992	143.717
1993	145.450
1994	141.750
1995	145.183
1996	147.200
1997	146.383



This information was obtained from Mini Tab.

Lesson 2

Preassessment – The teacher will begin the lesson by distributing an opening activity worksheet (Opening Activity Day 2 Worksheet). The purpose is to determine if students understood the material from Day 1 and can enter lists and create scatter plots and linear regressions on their own.

Launch – The teacher should assign students to groups of three or four. Each group should be given one rubber ball, a tape measure, several pieces of tape to attach the tape measure to the wall, and a worksheet.

Teacher Facilitation – Students should be instructed to move to an area with a hard floor. They should attach the tape measure to the wall vertically. Students should drop the ball from the heights given in the worksheet, and record the height of the first bounce.

Student Application – Using the data which the students recorded on the worksheet, they will create a scatter plot. They will look for outliers. They will then enter the data into their calculators, and calculate the linear regression model. They will use the correlation coefficient to determine the strength and direction of the association.

Embedded Assessment – Teacher will frequently check student progress.

Reteaching/Extension – For a home assignment the students should be directed to enter the Olympic Discus data into their calculator.
They should:

- create a scatter plot, and identify any outliers.
- calculate the linear regression model.
- use the model to predict the winning distance for 1916.
- use words, symbols, or both to explain how they got the prediction for 1916.
- determine the correlation coefficient and strength of the association.

NAME _____ DATE _____

Opening Activity Day 2

1. Using your TI-83 plus calculator, enter the following data into lists L_1 and L_2.

X	Y
-5	-13
-4	-10
-2	-7
0	-4
.5	0
2	0
3	4
4	5

2. Create a scatter plot of the data on your TI-83 plus calculator. Show the result to your teacher.

3. Use your TI-83 plus to find the regression line and correlation coefficient. Round answers to 3 decimal places.

Regression equation:_____

Correlation coefficient: _____

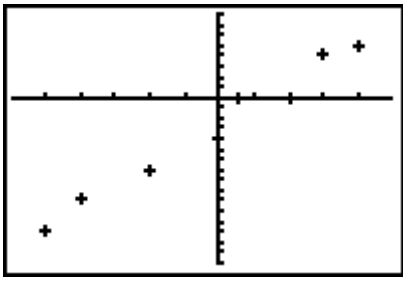
4. Graph the scatter plot along with the regression line on your TI-83 plus calculator. Show the result to your teacher.

Answer Sheet for
Opening Activity Day 2

1. Using your TI-83 plus calculator, enter the following data into lists L_1 and L_2.

X	Y
-5	-13
-4	-10
-2	-7
0	-4
.5	0
2	0
3	4
4	5

2. Create a scatter plot of the data on your TI-83 plus calculator.

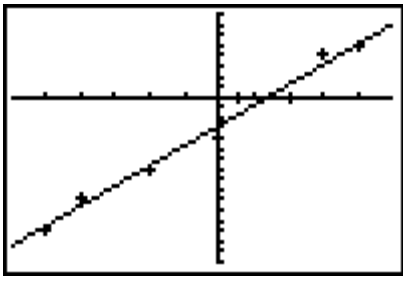


3. Use your TI-83 plus to find the regression line and correlation coefficient. Round answers to 3 decimal places.

Regression equation: **y = 1.978 x - 2.754**

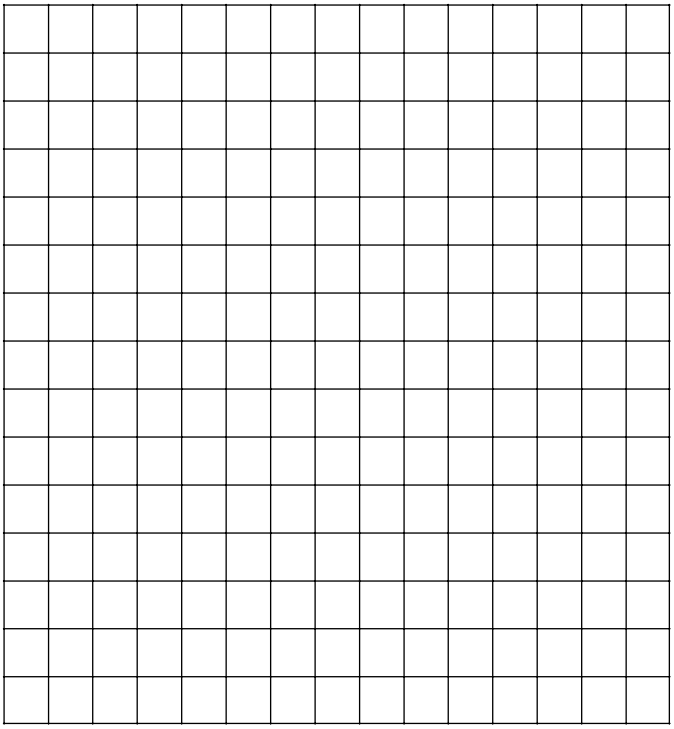
Correlation coefficient: **r = .987**

4. Graph the scatter plot along with the regression line on your TI-83 plus calculator. Show the result to your teacher.



Bouncing Ball Worksheet Day 2

Drop Height (in cm)	Bounce Height
100	
110	
120	
130	
150	
170	
180	
200	



Outliers? _____

Linear Regression Model: _____

Correlation coefficient: _____

Direction of association: _____

Strength of correlation: _____

Predicted height of a bounce dropped from 140 cm: _____

Predicted height of a bounce dropped from 250 cm: _____

Which of the above predictions should be more reliable? Justify your answer.

_____

_____

From what height should one drop the ball so that the bounce height will be 90 cm?

_____

Explain how you arrived at your answer to the previous question.

_____

_____

Lesson 3

Launch – The teacher will begin the lesson by dividing the students into groups of four to six students. Each group will need a tape measure, a pencil, and a sheet of paper. Teacher should instruct groups to measure their height and stride length or height and forearm length and record the results. To help with measuring stride length, note that teacher should pre-measure the length of the floor tiles.

The teacher will then describe the following scenario: Someone from this class has "borrowed" the principal's favorite pen. The person left behind his/her jacket and several muddy footprints as he/she left the office, so we know that his/her forearm length is 16 inches, and his/her stride length is 18 inches. How tall is the person who "borrowed" the principal's pen?

Based on forearm length or stride length, the students will determine which member of their group is most likely the person with the pen. The class will then compare the heights of all the suspects.

Teacher Facilitation – The teacher will assign students into groups of four to six. Students should be instructed to design their own experiments to collect bivariate data.

Student Application – Students should decide within their group the type of experiment they wish to perform, and the materials needed to perform it.

Embedded Assessment – At a later date, the teacher should provide class time for each group to perform their experiment, gather their data, and analyze results.

Teacher notes:

- This lesson will be split over at least two days.
- The experiment that students design should collect at least eight data values.
- Data does not need to be linear.
- If students need suggestions for experiments that should provide approximately linear data, the following may be used:
    1. Time vs. number of students in a chain: Students make a chain by gently squeezing the wrist of the person on their right. Measure the time from first squeeze to last squeeze. Add varying numbers of students to the chain.
    2. Forearm length vs. height: Students will measure their forearm length and height and see if there is a linear correlation.
    3. Weights on a spring: Suspend varying weights on a spring, and measure the spring length.
- A scoring rubric is included for this activity. Teachers may modify as needed.
- The teacher should distribute scoring rubric to students before they design their experiments.
- The launch activity was inspired by *Mathematics: Modeling Our World, Course 1*, UNIT 4, Prediction, developed by Comap, Inc., and published by W. H. Freeman and Company, 1998.

Scoring Rubric

3: Experiment is clearly and fully explained, and produced bivariate data. Data are displayed and organized in a neat and logical manner. Data are fully and appropriately analyzed, including all of the following: a scatter plot, a discussion as to whether or not the data are linear, and, if appropriate, the linear regression model and a discussion of the correlation coefficient.

2: Experiment is partially explained, and produced bivariate data. Data are completely displayed and organized. Data are partially analyzed, including some of the following: a scatter plot, a discussion as to whether or not the data are linear, and, if appropriate, the linear regression model and a discussion of the correlation coefficient.

1: Experiment is minimally explained, and produced bivariate data. Data are incompletely displayed. Data are minimally analyzed, including a scatter plot and a linear regression model.

0: Experiment did not produce bivariate data.

or

Experiment produces bivariate data, but is not displayed or analyzed.

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